Contributions to Rheological Fluids Flow in Modified Couette Device

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The liquids motion in ring-shaped spaces, based on the rotation of one or both coaxial cylinders, leads to the liquid layers relative motion with velocity values between those of lateral surfaces. The friction force and the shear rate depend on the assessed revolution, cylinders geometry and liquid dynamic viscosity. The Taylor number (Ta), defined as the ratio between the centrifugal force and the viscous friction one, characterizes the flow nature and indicates the possibility of specific effects appearance (Taylor-Couette instability). Based on these reasons it was optimized the dimensioning of a modified Couette rheometer (the inner cylinder is subjected to rotational motion and the outer is setting into an elastic joint accordingly to the developed friction forces), to allow the study of medium viscosity fluids behaviour (water, aqueous solutions, ethylene glycol, mineral oils), in transient domain (60<Ta<3000). The device construction offers the possibility to view the effects that appear, as well as the functional sizes acquisition (revolution, extension force, elastic motion) using sensors and interfaces and their further processing.

Keywords: rheometer, rotational motion, Taylor number

For liquids subject to rotational motion (stirring in cylindrical vessels, centrifugal pumps or liquid-ring pumps, extractors, rotational viscometer, lubricating bearings), the hydrodynamic criteria Reynolds, Re and Euler, *Eu* have characteristic expressions as a function of: the liquid properties (density ρ, dynamic viscosity η, or kinematic viscosity $\nu = \eta / \rho$), the dimensions of rotating system (radius *r*, the thickness of the ring-shaped space Δr), the rotational motion intensity (revolution *n*, angular velocity Ω), the power used to achieve the motion *P*:

$$\mathrm{Re}=n\cdot d^2\cdot\rho/\eta=n\cdot d^2/\nu \quad ; \qquad Eu = P/n^3\cdot d^5\cdot\rho \qquad (1a,b)$$

The characterization of liquids flow in ring-shaped spaces is expressed using the Taylor number, T_a defined as the ratio between the centrifugal force and the viscous friction one:

$$Ta=F_{centrifugal}/F_{visccous}= 4\cdot\Omega^2\cdot R^2/\nu^2 \quad ; \qquad (2)$$

Another method to characterize the hydrodynamic regime is the Taylor-Reynolds number, $T_{a\mathrm{Re}}$ which depends on the geometry of the ring-shaped space (radii of the two cylinders), angular velocity and viscosity [1-3]:

$$Ta_{\mathrm{Re}}=\frac{\Omega\cdot(r_o^2-r_i^2)}{2\cdot\nu}=\frac{\pi\cdot n\cdot(r_o^2-r_i^2)}{\nu} \qquad \text{or} \qquad Ta_{\mathrm{Re}}=\frac{\Omega\cdot r_i\cdot(r_o-r_i)}{\nu} \qquad (3a,b)$$

A special interest presents the system behaviour when is overtaken the laminar flow regime, especially the intimation of flow instability phenomena. The critical value if this regime is $T_{a\mathrm{Re}}$=60.

Rotational rheometers are based on the effect measurement made by the viscous forces which are developed into a fluid placed into the ring-shaped space of two coaxial cylinders. One or both of them execute controlled rotational motions. Thanks to the difference between the velocities of successive fluid layers subjected to rotational motion, it is possible to establish the shear rate $\gamma = dw / dy$. The flow and friction effect between the layers leads to a torsion effort appearance in the other cylinder. Through its measurement it is calculated the shear stress $\tau = F_f / S_l$ (F_f - the friction force and S_l-the contact surface between the cylinder and the liquid). Functional, it is possible to appear the followed situations:

- the outer cylinder is subjected to rotational motion and the torsion effect is measured to the inner one (original Couette viscometer) [4];
- the inner cylinder is turn round and the torsion effect is measured through the partial rotation of the outer cylinder (modified Couette viscometer);
- the inner cylinder is subjected to rotational motion and is measured its torsion moment; the outer cylinder is rigid (Rheotest rheometer);
- one of the cylinders is subjected to rotational motion with an angular velocity Ω_1 and the other one is turnround with a smaller forces through the fluid (Taylor-Couette rheometer).

Through the shear stress correlation with the shear rate, $\tau = f(\gamma)$ it is possible to establish the rheological behaviour of different fluid types.

Experimental part

The modified Couette rheometer proposed to achieve and experiment is based on the followed principles, figure 1:

-the inner cylinder A is turnround with angular velocity Ω;

- the outer cylinder B is setting into an elastic joint;

-the torsion moment transmitted to the outer cylinder is proportional to τ;

-the prescribed data is the angular velocity Ω;

-the measured data is the torsion angle θ of the outer cylinder which is correlated with the shear stress τ;

- Ω and τ are measured on different axes;

By rotation of the inner cylinder, the liquid placed in the ring-shaped space is set in motion. Due to the friction between the liquid and the outer cylinder wall a torsion force is turned up. This one formed a rotational motion to the outer cylinder in the same direction with the inner one.

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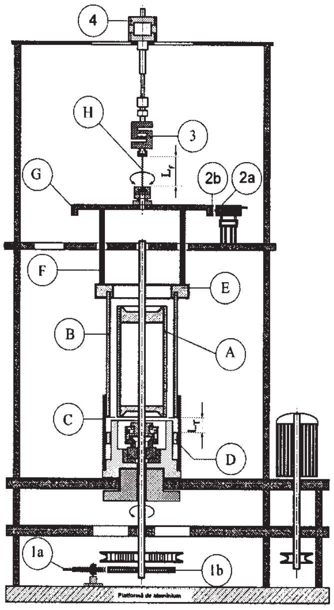

Fig.1. Couette rheometer with modified measured system

Through the elastic setting of the outer cylinder, once the beginning of its circular motion, into the elastic joint appears a reaction elastic force F_e. The torsion force F_t has maximum value at the beginning of motion and tends to zero as the elastic force increases from zero to the maximum value.

In the first stage of the motion cycle, the outer cylinder rotation has the same direction with the fluid and the inner one and reaches the equilibrium state between the torsion force F_t^* and the elastic one F_e^*. Due to inertia, the outer cylinder is follow-up in rotational motion until the point of maximum position where the torsion force F_t is cancelled by the elastic force F_e. The value of rotational motion amplitude, in the first stage of the motion cycle, is represented by the proper central angle θ_1 by which the outer cylinder is moved between the initial and the maximum position.

In the second stage of the motion cycle, the outer cylinder rotation takes place in opposite direction in comparison with the inner one, crosses through an equilibrium position of torsion and elastic forces and due to inertia is follow-up in move until a minimum position. In this point, the torsion force is cancelled again by the elastic one. According to the rotation from the second stage of the motion cycle, the amplitude will be represented by the central angle $\theta'_1(\theta'_1<\theta_1)$.

The oscillations cycle (partially circular motions, forward and back) of the outer cylinder is repeated until this one attains a stationary position which is phase-shifted in comparison with the initially one with a central angle $\Delta\theta_{ech}$. This behaviour is considered as an amortized oscillating motion. For a certain hydrodynamic flow regime, the value of the central angle $\Delta\theta_{ech}$ depends on the fluid nature placed in the ring-shaped space, more exactly on its viscosity.

Results and discussion

For apparatus dimensioning it were accepted the internal radius r_i and the thickness of ring-shaped space

Table 1a
THE REVOLUTION VALUES AT DIFFERENT T_{aRe} VALUES (WATER)

Nr.	r_0 [mm]	$r_0 - r_i$ [mm]	Revolution, n [s^{-1}]				
			Ta_{Re}=10	Ta_{Re}=60	Ta_{Re}=180	Ta_{Re}=2000	Ta_{Re}=3000
r_i = 50 mm							
1	51	1	0.0315	0,189	0,567	6,30	9,46
2	52	2	0.0156	0,094	0.281	3,12	4,68
3	53	3	0.0103	0,062	0,185	2,06	3.09
4	54	4	0,0077	0.046	0,138	1.54	2.30
r_i =40 mm							
1	41	1	0,0393	0,236	0,708	7,87	11,80
2	42	2	0,0194	0,116	0,348	3,89	5,83
3	43	3	0,0128	0,077	0,231	2.56	3,84
4	44	4	0,0095	0,057	0,171	2,00	2,84
r_i = 30 mm							
1	31	1	0.052	0,313	0.93	10,4	15,7
2	32	2	0,026	0,154	0,46	5,13	7.70
3	33	3	0,017	0,101	0.30	3,37	5.06
4	34	4	0,012	0,074	0,22	2,48	3,73

Table 1b
THE REVOLUTION VALUES AT DIFFERENT T_{aRe} VALUES (ETHYLENE GLYCOL)

Nr.	r_0 [mm]	$r_0 - r_i$ [mm]	Revolution, n $[s^{-1}]$				
			$Ta_{Re}=10$	$Ta_{Re}=60$	$Ta_{Re}=180$	$Ta_{Re}=500$	$Ta_{Re}=1000$
r_i = 50 mm							
1	51	1	0.651	3,91	11,7	32,6	65,1
2	52	2	0.322	1.93	5,80	16,1	32.2
3	53	3	0.213	1,28	3,83	10,6	21.3
4	54	4	0,158	0,95	2,85	7,90	15.8
r_i =40 mm							
1	41	1	0,81	4,87	14.6	40.6	81.2
2	42	2	0,40	2.40	7.22	20.0	40.1
3	43	3	0.26	1.58	4.75	13.2	26.4
4	44	4	0,19	1,17	3.52	9.78	19.6
r_i = 30 mm							
1	31	1	1,08	6,46	19,4	53,9	108
2	32	2	0.53	3,18	9.54	26,5	53.0
3	33	3	0,35	2.09	6.26	17.4	34,8
4	34	4	0,26	1,54	4.62	12.8	25,7

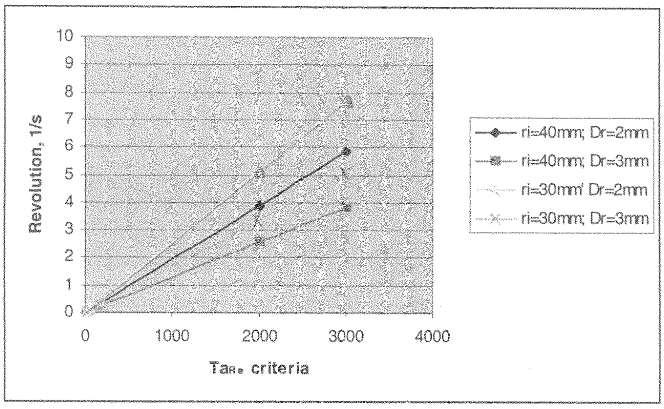

Fig. 2. Revolution versus Ta_{Re} criteria for water

$\Delta r = r_o - r_i$ and were established the necessary revolution values to achieve the inferior and superior limit of the intermediate flow regime in ring-shaped space (T_{aRe} = 60 and T_{aRe} = 3000). Also, it were performed calculations for other T_{aRe} values: $10 \le T_{aRe} \le 3000$ (for water) or $10 \le T_{aRe} \le 1000$ (for a more viscous fluid, ethylene glycol).

$$n = Ta_{Re} \cdot \frac{\eta}{\pi \cdot \rho \cdot \left(r_o^2 - r_i^2\right)} = \frac{Ta_{Re} \cdot \nu}{\pi \cdot \left(r_o^2 - r_i^2\right)} \tag{4}$$

For: r_i = 50; 40; 30 mm and Δr = 1; 2; 3; 4 mm, the revolution values, at different T_{aRe} values, are presented in table 1a and 1b:

A. Water:

$\rho = 1000\ kg \cdot m^{-3}$; $\eta = 10^{-3}\ kg \cdot m^{-1} \cdot s^{-1}$; $\nu = 10^{-6}\ m^2 \cdot s^{-1}$.

B.Ethylene glycol:

$\rho = 1114\ kg \cdot m^{-3}$; $\eta = 23 \cdot 10^{-3}\ kg \cdot m^{-1} \cdot s^{-1}$; $\nu = 2{,}065 \cdot 10^{-5}\ m^2 \cdot s^{-1}$

For water, the obtained results are presented in figure 2.

Conclusions

Using water, the attainment of the intermediate flow regime (T_{aRe}=60) is possible at very small revolution values. For ethylene glycol, the values are about 20 times bigger.

For a certain value of the flow regime, the revolution is doubled when the inner radius and the thickness of the ring-shaped space are half reduced.

The larger the surface between the liquid and the outer cylinder, the more sensitive the apparatus: $S_l = \pi \cdot d_e \cdot H_l$, where H_l is the useful height of the liquid column. The diameter decreasing to increase the revolution leads to the growth of cylinder height.

In order to perceive specific instability phenomena for the transition point between laminar and intermediate flow (T_{aRe}=60) and the rotation velocity of the inner cylinder be adequate, it is necessary to take into account the following:

- external diameter needs to be as small as possible. For usual viscometers it is recommended that the diameter to be approximately 40mm. Due to the production costs increasing with the apparatus miniaturizing, for the experimental one are permitted superior values (80mm);

- useful height H_l needs to be as large as possible. The correlation between the inner diameter and the cylinder height H_c is achieved using the slenderness ratio $c_{sl}=d_t / H_c$. The smaller this coefficient, the smaller the influence of the end effect about the measurement accuracy. As optimal value was considered $c_{sl}=1/ 6$.

- thickness of the ring-shaped space needs to be as small as possible. For its characterization are used both the radii difference $\Delta r = r_o - r_i$ and their ratio $\delta = r_o / r_i$ (it is recommended that $1.01\leq \delta \leq 1.1$). For Newtonian fluids the ratio tends to the superior limit $\delta=1.1$, for special rheometer $\delta =1.003$. For the characterization of suspensions or dispersions with solid particles or gases inclusion it is recommended that Δr to be three times bigger than the particle diameter.

Constructive solutions

The viscometer cylinders were glass made, taken into consideration the smaller roughness and price in comparison with steel. Also, the corrosion resistance of the glass tubes is very high in comparison with other metals like copper and lead, of which it is possible to manufacture pipes with a roughness comparable with the glass. The use of the glass allows to visualize the phenomena which appear during the flow in the ring-shaped space (ex. a multiphase fluid flow).

The disadvantages of using glass tubes for viscometer cylinders consist in: a) the tubes present a certain out-of-roundness; b) the fixation and centering upon a rotation axis is difficult and need expensive technical solutions.

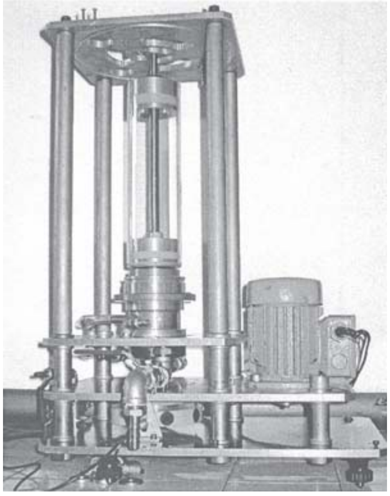

Fig. 3a. Couette rheometer with modified measured system. General view

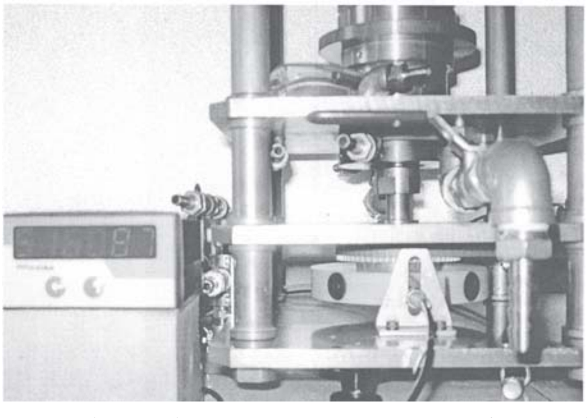

Fig. 3b. The inner cylinder turnround and the system for revolution

Taking into account the anterior explanations and the results obtained through calculation, there were chosen Duran type glass tubes (Schott) with following characteristics:

-external diameter of the inner glass cylinder $d_i=80$ mm;

-internal diameter of the outer glass cylinder $d_o=84$ mm

There were obtained the thickness of the ring-shaped space $\Delta r = r_o - r_i = 2$ mm and the ratio $\rho = r_o/r_i = 1.05$. For the height it was considered as the base of calculation the inner cylinder which corresponds to the liquid column height from the ring-shaped space. As constructive solution was admitted $H_l = H_c = 290$ mm and a slenderness ratio $c_{si}=d_i / H_c = 40 / 290 = 0.138$

.

Details of some components

As already stated, the experimental equipment consists of two coaxial glass cylinders; the inner cylinder is subjected to continuous rotation, and the outer one is mounted by means of an elastic connection, which allows for partial rotation under a certain angle around its centre $\Delta\theta$. The connection of the outer cylinder is set at the bottom by means of an elastic tube (rubber) and, at its top, it is centered with the inner cylinder and their common axis by means of a guide. Technical solutions have been devised for the guide as a mechanical one (circular with air cushion, non-friction, or a radial-axial bearing respectively), or as an axial elastic guide which consists of an elastic tube placed under the measuring disc (if it is possible to support it), or above the disc, in which case it is necessary to strain it by an elastic string. The latter version provides control of the centering of the cylinders, and the straining of the string does not produce vibrations. In figure 1, the automated version of the viscometer is described with drawn axial elastic guide, where the adjustment of the string straining is done by means of a control loop consisting of a stepper motor and a force probe. The rheometer achievement is shown in figures 3a,b,c.

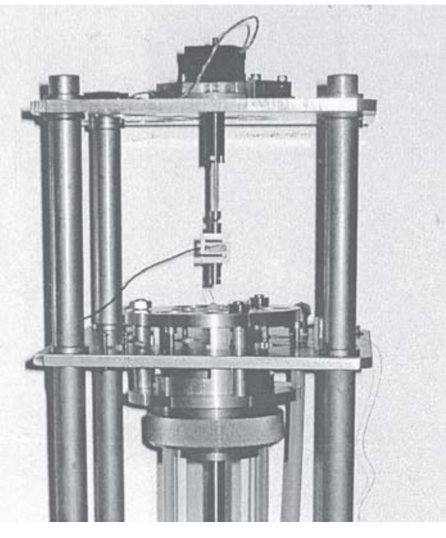

Fig. 3c. Viscometer with axial elastic guide

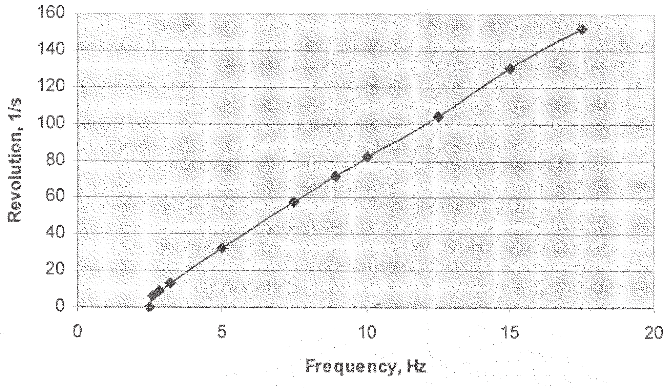

Fig. 4. The dependence of revolution values on frequency

The glass coaxial cylinders and the solution feeding tank are placed in a thermostated container, cylindrical in shape, made of a transparent material. The cover of the container is provided with orifices which allow the entering/evacuation of the thermostated water, control of temperature and setting of the position of the working solution feeding tank.

The filling and emptying of the ring-shaped space with the work solutions kept in glass tanks placed in the thermostated container is done at its bottom, through a set of spherical valves and tubes respectively, through a bevel cut in the packing gland.

The driving of the system is done by means of an electric motor whose rotation speed is variable by modulating the frequency (1÷120 Hz) of the input current. It is possible to achieve a theoretical revolution n = 22 s^{-1}. The transmission of motion from the motor to the shaft of the inner cylinder is done through a dented belt. The two dented wheels of the motor shaft and of the inner cylinder shaft are in a rate of 1:3 (reducing). The dependence between the measured revolution and frequency for this system is shown in figure 4.

The sealing of the ring-shaped space is done by means of a sliding system having circular plane surfaces. A ceramic ring affixed in the packing gland body is the fixed part, and a carbon ring is mounted on the driving shaft of the inner cylinder. The mounting of the glass external cylinder on the bottom side of the thermostated container is done through the elastic tube whose bottom side is fixed by a threaded ring on the external surface of the packing gland, which allows for adjusting the length of the elastic tube.

The value of angle $\Delta\theta_{ech}$ which reflects the rotation of the external cylinder from its initial position is detected and transformed into an electric signal unified by means of a motion measuring system, affixed onto a disc which makes common body with the outer glass cylinder and with the fixed part of the viscometer respectively. The unified electric signal may be amplified and modulated in such a way as to allow its analysis by an oscilloscope or to be graphically recorded.

The connection to a computer is enabled through two active RS232 interfaces, for the control loop of the elastic gasket straining (the force probe and the stepping motor), and for measuring the rotation speed of the inner cylinder and of the circle arc by which the outer cylinder moves, respectively.

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